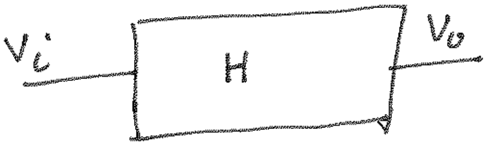
Experiment 5
Filters

- A filter is a frequency selective device. It allows certain frequencies to pass almost unattenuated within the passband and rejects (suppresses) other frequencies within the rejection band.

- The decibel concept



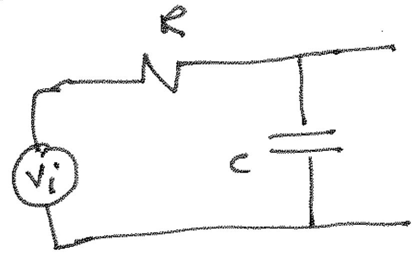
The attenuation or (gain) of a network is

$$G = \frac{V_o}{V_i}$$

in logarithmic, called the decibel, terms, the gain(attenuation) is defined as

$$G_{dB} = 20\log_{10} \frac{V_o}{V_i}$$

- The First order low pass Filter



Let $v_i(t) = \sqrt{2}\, V_i \cos 2\pi f t$

In phasor term, it is expressed as $V_i \angle 0$

The transfer function of the network is

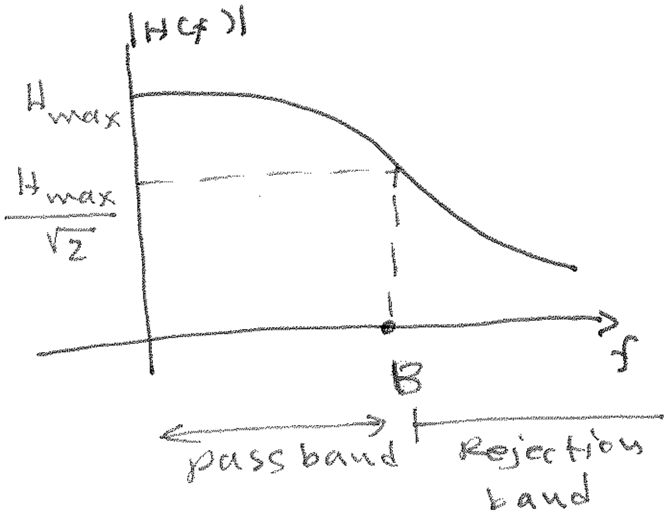
$$H(f) = \frac{1/j2\pi f C}{R + 1/j2\pi f C} = \frac{1}{1+j2\pi f RC} = \frac{1}{\sqrt{1+(2\pi f RC)^2}} \angle -\tan^{-1} 2\pi f RC$$

(2)

- The magnitude of $H(f)$ is

$$|H(f)| = \frac{1}{\sqrt{(2\pi f RC)^2 + 1}}$$

and is plotted below



$H(0) = 1$

$$H\left(f_c = \frac{1}{2\pi RC}\right) = \frac{1}{\sqrt{2}}$$

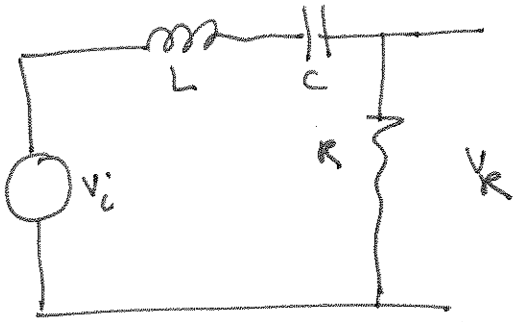
$f_c = \frac{1}{2\pi RC}$ is called the 3-dB bandwidth

Note that $20 \log \frac{H(f_c)}{H_{max}} = 20 \log_{10} \frac{1/\sqrt{2}}{1} = -20 \log \sqrt{2}$

$= -3\text{ dB}$

Second order Filter

The filter transfer function



$$H_R(f) = \frac{V_R}{V_i} = \frac{R}{R + j\omega L + 1/j\omega C}$$

$$H(f) = \frac{\omega RC}{\omega RC + j(\omega^2 LC - 1)}$$

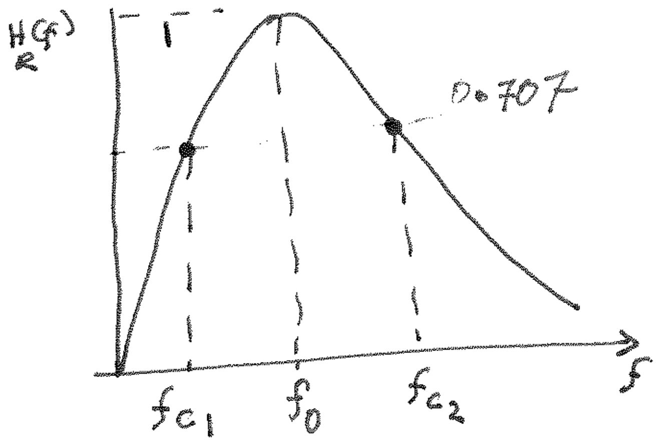
$$|H(f)| = \frac{\omega RC}{\sqrt{(\omega RC)^2 + [(\omega^2 LC - 1)]^2}}$$

critical points when $\boxed{(\omega RC) = [\omega^2 LC - 1]}$ solve for f_{c1} and f_{c2}

f_0 : Resonance frequency

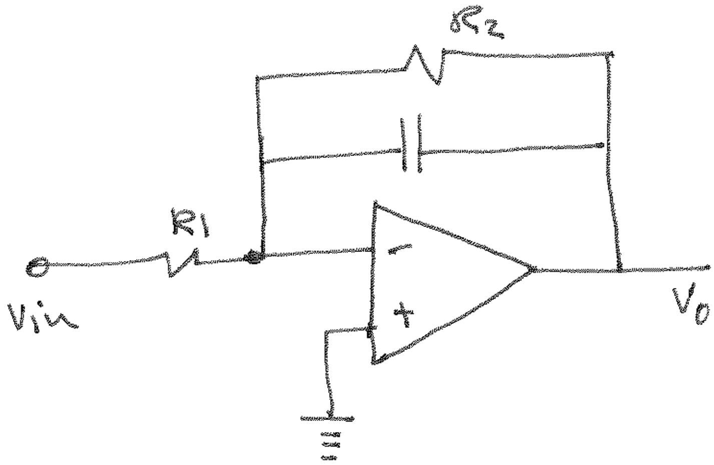
$\omega_0^2 LC = 1$ or $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Filter bandwidth $= f_{c2} - f_{c1}$

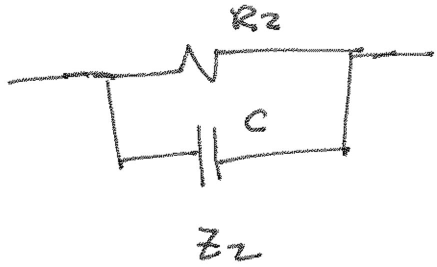


Active Filter

The transfer function of the inverting OP-AMP is



$$H(f) = \frac{V_o}{V_i} = -\frac{1}{R_1} Z_2$$



$$Z_2 = R_2 \| (1/j\omega C)$$

$$= \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}}$$

$$= \frac{R_2}{1 + j\omega R_2 C}$$

$$H(f) = \frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C}$$

$$|H(f)| = \frac{R_2}{R_1} \cdot \frac{1}{\sqrt{1 + (\omega R_2 C)^2}}$$

3-dB point when $\omega_c R_2 C = 1 \Rightarrow f_c = \frac{1}{2\pi R_2 C}$

Note that $|H(f=0)| = \frac{R_2}{R_1}$